ARISTOTLE, MENAECHMUS, AND CIRCULAR PROOF

I: Circular Proof

(a) *The Regress*: Knowledge, we like to suppose, is essentially a rational thing: if I claim to know something, I must be prepared to back up my claim by stating my reasons for making it; and if my claim is to be upheld, my reasons must be good reasons. Now suppose I know that Q; and let my reasons be conjunctively contained in the proposition that R. Clearly, I must *believe* that R (for R cannot give my reasons unless it has my assent); equally clearly, I must *know* that R (for mere opinion is not nutritious enough to sustain the demanding body of knowledge). Thus if I know that Q, I know that R. But if I know that R, then I must have my reasons, R', for holding R; and, by the same argument, I must know that R'. And if R', then R''; and so on, *ad infinitum.*

Thus it appears that if I know anything at all, I know infinitely many things: an interminable regress lies open before me.

The dangers, real or imagined, that such an argument threatens have determined the course of many an epistemological progress: the regress was adverted to in Plato's *Theaetetus*; and it is still the object of lively philosophical controversy.[1] A topic of vigorous debate in the Academy,[2] it was discussed and given a classic answer by Aristotle in his *Posterior Analytics*.

Aristotle attempts to avoid the regress by denying the fundamental assumption on which it depends; for he asserts, in effect, that there is a type of knowledge which is not rational or based on reasons. In the version of the regress which Aristotle considers, the assumption is that all knowledge is demonstrative: if I know something, my reasons must have probative force; if I know that Q, then I have demonstrated Q from known premisses R. Aristotle rejects the assumption: '. . . *we* say that neither is knowledge (ἐπιστήμη) demonstrative, but in the case of the immediates it is non-demonstrable . . .' (*APo.* A 3, 72^{b} 18–20). That brusque statement is not elaborated in *APo.* A 3: the bulk of the chapter examines and criticizes an alternative attempt to evade the regress; and it is that un-Aristotelian attempt which forms the subject of my paper.

(b) *The Regress Refined*: It will simplify things if we consider a single knower or 'scientist' a, and a single branch of knowledge, or 'science' (ἐπιστήμη), S. The constituent propositions of S – the truths of the science – can be set out in a sequence, thus:

$$S = \ldots, P_n, P_{n+1}, P_{n+2}, \ldots, P_{n+m}, \ldots$$

[1] See *Tht.* 209 e – 210 b. (The regress has also been connected with the matter of Socrates' dream at 201 d – 202 d: see G.R. Morrow, 'Plato and the Mathematicians: an Interpretation of Socrates' Dream in the *Theaetetus*', *Philosophical Review*, 79 (1970), 309–33.) For modern discussion see, e.g., M. Deutscher, 'Regresses, Reasons and Grounds', *Australasian Journal of Philosophy*, 51 (1973), 1–16; D.M. Armstrong, *Belief, Truth and Knowledge* (Cambridge, 1973), pp.150–61 (of the six reactions to the regress which Armstrong lists, Aristotle considers and rejects (1) and (3), and accepts a version of (4)); N.M.L. Nathan, 'Scepticism and the Regress of Justification', *Proceedings of the Aristotelian Society*, 75 (1974/5) 77–88.

[2] In addition to Arist. *APo.* A 3 and the texts discussed below, see *Metaph.* A 2; Theophrastus, *Met.* 9^{b} 21; cf. Sextus, *adv. Math.* 8. 347.

The regress depends, in Aristotle's presentation, on two premisses. The first is expressed at *APo.* A 3, 72b 8: *ὑποθέμενοι μὴ εἶναι ἄλλως*[3] *ἐπίστασθαι* . . . , 'supposing that it is not possible to know otherwise < than by demonstrating>'. Thus *a* will only know a truth of *S* if he has demonstrated it:

(1) For any *i*: if *a* knows that P_i, then *a* has demonstrated that P_i.

The second premiss is implicit in 72b 9: *ὡς οὐκ ἂν ἐπισταμένους τὰ ὕστερα διὰ τὰ πρότερα* . . . , 'we would not know the later <propositions> on the basis of the earlier ones'. Thus if *a* does know a truth of *S*, his knowledge rests on some earlier truth of *S*; and that earlier truth must be known by *a* and used by him as a premiss of his demonstration. What that amounts to is:

(2) For any *i*: if *a* has demonstrated that P_i, then there is some *j* smaller than *i* such that *a* knows that P_j and *a* has inferred P_i from P_j.

From (1) and (2) there immediately follows:

(3) For any *i*: if *a* knows that P_i, then there is some *j* smaller than *i* such that *a* knows that P_j and *a* has inferred P_i from P_j.

Now (3) says that if I know anything, then I have inferred it from some prior piece of knowledge; and that surely means that if I know anything, I have made infinitely many inferences and know infinitely many things.

Two regresses, not one, have appeared – a regress of knowledge and a regress of inference. The former at least is not necessarily vicious; after all, anyone acquainted with elementary arithmetic possesses a strictly infinite store of knowledge. But the inferential regress is a different matter; for if we are creatures of a finite lifespan, and if there is an upper limit to our speed of inference (no act of inferring, say, can take less than a micro-second), then we do not have the capacity to make an infinitude of inferences. Although Aristotle does not distinguish the two regresses, his use of the dynamic term *διελθεῖν* at 72b 11 perhaps indicates that he had the inferential regress primarily in mind.

How should we react to the regress? Some sceptical philosophers (*οἱ μέν*: 72b 7) maintained that (1) and (2) were both true; and finding the possession of infinite intellectual capacities an impossibility, they concluded that 'there is no knowledge' (72b 6). Other philosophers (*οἱ δέ* : 72b 15) and Aristotle himself (*ἡμεῖς* : 72b 18) agreed with the sceptics on the impossibility of intellectual infinitude, but staunchly affirmed the existence of knowledge: Aristotle did so by denying (1); *οἱ δέ* , however, wanted to retain that premiss (72b 16).

(c) *Circular Demonstration*: *οἱ δέ*, according to 72b 16–18, maintain their position by urging that 'nothing prevents there being demonstration of everything; for it is possible for the demonstration to come about circularly and reciprocally (*κύκλῳ* . . . *καὶ ἐξ ἀλλήλων*)'. What does this mean? and how is it designed to stave off scepticism?

The circular demonstrators are out to vindicate knowledge; they assert, therefore, that some things are known – for some *i*, *a* knows that P_i. They also maintain (1), and hence hold that knowing involves inferring; yet they reject any infinite inferential capacities. Hence *a* has only finite knowledge; and we may construct a finite subset, S^*, of *S* by ascribing P_i to S^* if and only if *a* knows that P_i.

[3] This, and not ὅλως, is the right reading (cf. 72b 12; 16) – *pace* W.D. Ross, *Aristotle's Prior and Posterior Analytics* (Oxford, 1949), p.514 (hereafter *APPA*).

Given (1) if follows trivially that all members of S^* have been proved by a:

(4) For any i: if P_i is in S^*, then a has demonstrated that P_i.

The circular dictum that 'nothing prevents there being demonstration of everything' amounts to the assertion that (4) describes a logically possible situation. The premisses used in a's demonstrations must, of course, be truths known by a; hence they will be members of S^*. Thus:

(5) For any i: if P_i is in S, then there is some j such that P_j is in S and a has inferred P_i from P_j.

Now the relation of *being inferred from* is transitive (if P is inferred from Q and Q from R, then P is inferred from R); and S^* is finite; hence it is easy to see that:

(6) For some i and some j: P_i is in S^* and P_j is in S^* and a has inferred P_i from P_j and a has inferred P_j from P_i.

And the *dictum* that 'it is possible for the demonstration to come about circularly and reciprocally' amounts to the assertion that (6) describes a logically possible situation.

(d) *An Initial Criticism*: At *APo.* A 3, 72b 25 - 73a 20, Aristotle assembles three arguments to show that 'it is both empty and impossible to say that demonstration is reciprocal and that because of this there can be demonstration of everything' (73a 18-20).[4] He first argues that circular demonstrators must hold that 'the same things at the same time <are> prior and posterior to the same things' (72b 27). The argument is based on premiss (2); for if j is smaller than i, then P_j is prior to P_i, and P_i is dependent upon P_j (see *APo.* A 2, 71b 22; 71b 31 - 72a 5). Given (2), it is plain that the circular demonstrator is committed not just to (6) but also to:

(7) For some i and some j: P_i is in S^* and P_j is in S^* and j is smaller than i and i is smaller than j.

And (7) is contradictory.

But Aristotle's argument betrays *ignoratio elenchi*: it relies upon (2); but the circular demonstrator does not adhere to (2) – indeed, it is precisely by rejecting (2) that he hopes to escape from the regress. Aristotle's criticism imputes to the circular men the very proposition which they are determined to reject. They do not require that S, or S^*, be ordered by Aristotelian priority; instead of (2) they will offer:

(8) For any i: if a has demonstrated that P_i, then there is some j distinct from i such that a knows that P_j and a has inferred P_i from P_j.

The premisses of a proof must be distinct from its conclusion; but they need not be prior to it.

According to Aristotle (A 2, 71b 31), each P_i must give the 'cause' or explanation (*αἴτιον*) of its successor in S; and it is the asymmetrical relation of 'causation' which accounts for the appearance of priority (in the guise of 'j is smaller than i') in (2). But some of Aristotle's contemporaries argued that the notion of 'causation' is quite alien to those paradigm sources of demonstrative reasoning, the mathematical sciences:[5] if demonstrations do not need 'causes',

[4] I discuss here only some aspects of Aristotle's first argument; for further notes on *APo.* A 3, see J. Barnes, *Aristotle: Posterior Analytics* (Oxford, 1975), pp.106–12 (hereafter *APA*).

[5] The Academic Amphinomus took this line: see the discussion in Proclus, *Comm. in Eucl.* 202.9–25; cf. T.L. Heath, *Euclid's Elements*² (Cambridge, 1926), i. 150, n.1 (hereafter *E*). For a clear illustration of Aristotle's view see *EE* 1222b 31–7 (discussed in T.L. Heath, *Mathematics in Aristotle* (Oxford, 1949), pp.279–80 (hereafter *MA*).

they do not need priority; and (8) is an adequate substitute for (2).

(e) *An Anachronistic Sophistication*: We might suggest to the circular reasoners a somewhat more subtle move. Any science is susceptible to more than one axiomatization. Circular proof, then, need not imply that P_i is both prior and posterior to P_j '*at the same time*' (72^b 27), even if premiss (2) is maintained; for P_i may be prior to P_j 'at one time' or under one axiomatization, and posterior to P_j 'at another time' or under a different axiomatization. Premiss (2), in other words, needs to be presented in the sophisticated form of:

(9) For any i: if a has demonstration that P_i , then there is some j and some axiomatization A such that P_j is prior to P_i in A and a knows that P_j and a has inferred P_i from P_j in A.

Whereas (8) completely undercuts Aristotle's criticism, (9) permits a modified form of the argument. But in that form it concludes not to (7) but to:

(10) For some i and some j: P_i is in S^* and P_j is in S^* and for axiomatizations A and A' P_i is prior to P_j in A and P_j is prior to P_i in A'.

There is nothing contradictory about (10).

(f) *A Better Criticism*: Premiss (2) contains the asymmetrical relation of priority; and Aristotle's first objection to circular proof unfairly relies on that fact. Having moved from (2) to (8), the circular reasoners hope to find their new accommodation free from any tiresomely asymmetrical relations. But in fact (8) contains, implicitly, a priority relation; and, following Aristotle's sound instinct, we may use that fact to hang the circular men.

If a's knowledge that P_i rests on an inference from a known P_j, then P_j must be known to *a beforehand*. In demonstrations, as Aristotle explicitly recognizes (*APo.* A 1, 71^b 31-3), the premisses must be known before the conclusion is inferred.[6] Proposition (8), then, contains implicitly the asymmetrical relation of *being earlier than*. The fact can be made explicit by introducing the relation into (8) at the appropriate point, thus:

(8a) For any i: if a demonstrates at time t_o that P_i, then there is some j distinct from i and some time t earlier than t_o such that a knew at t that P_j and a did not know at t that P_i and at t_o a infers p_i from P_j.

But now it is easy to saddle the circular reasoners with a contradiction: Aristotle rested his argument on the clause 'j is smaller than i' in (2); we may recast his argument so that it relies on the clause 't is earlier than t_o' in (8a). If my knowledge of P_i is derived from my knowledge of P_j, and vice versa, then I knew P_j before I knew P_i, and vice versa. And that is absurd.

The circular theorists cannot get away from priority; and that fact, as Aristotle saw, is enough to bring their theory down.[7]

II. Menaechmus

(a) *The Geometers*: We cannot put a name to the sceptical οἱ μέν; nor indeed need we suppose that Aristotle had in mind any named individuals as advocates

[6] *APo.* A 1, 71^a 17-21, argues that not *all* the premisses of a demonstration need be known before the inference is drawn. Aristotle's argument is faulty (see Barnes, *APA*, p.93); but even so he can still put forward the view that I construct for him – for he insists that at least *some* of the premisses must be known beforehand.

[7] A little misplaced ingenuity will in fact revive the theory; and the sophisticated version of section (e) is not yet dead. But the remaining breaths are few and not worth chronicling.

of scepticism.[8] Circular proof is another matter; for although no philosopher has a patent on the practice of circular reasoning, it is one thing to indulge in a circular argument and quite another to commend circular argumentation as a reputable form of scientific proof. We are justified in asking which theorists did commend it, and in trying to affix a name to Aristotle's anonymous *οἱ δέ*.[9]

Many scholars stress logic's debt to mathematics, and hold that the *Analytics* drew its inspiration from the great formal achievements of early Greek geometry.[10] Did any of those early geometers suggest the method of circular proof?

Three straws indicate that the wind blew from that quarter. First, the mathematicians who found no place for 'causes' in their studies made way for the rejection of (2) in favour of (8); and hence they made way for circular proof. Secondly, *APo.* A 3 stresses that circular proof requires convertible premisses; and elsewhere Aristotle correctly observes that 'propositions in mathematics convert more, because they assume nothing incidental' (A 12 78^a 10).[11] Thirdly, in *APr.* B 16 where Aristotle discusses *petitio principii*, he takes as an illustration 'those who think they can prove parallels' (65^a 4). He is referring, it seems, to some geometrical attempt to prove what was to become Euclid's fifth postulate;[12] and he says that 'those who argue in this way manage to say that each thing is the case if it is the case – but in this way everything will be knowable through itself, which is impossible' (65^a 7-9). There are close affinities between this passage and the second of Aristotle's objections to circular proof in *APo.* A 3, 72^b 32 - 73^a 6: some geometers of Aristotle's acquaintance were indulging, perhaps deliberately, in circular proof.

8 Ross, *APPA*, pp. 513-14, compares *Metaph.* 1011^a 3-13; 1006^a 5-9; 1012^a 20-1. In these passages, and hence in *APo.* A 3, he sees an allusion to Antisthenes (following H. Maier, *Die Syllogistik des Aristoteles* (Tübingen, 1896-1900), vol. ii b, p.15, n.2). Similarly H.F. Cherniss, *Aristotle's Criticism of Plato and the Academy* (Baltimore, 1944), pp. 64-6 (hereafter *ACPA*) – though Cherniss rightly adds that 'anyone who was inclined to scepticism would be likely to maintain' the position Aristotle describes. The *Metaphysics* passages do not seem to me to deal with the same issue as *APo.* A 3 (I note in passing that T. Waitz, *Aristotelis Organon* (Leipzig, 1844-6), ii. 310, cites two of them as parallels to the thesis of the circular reasoners); nor are Maier's reasons for attaching the *Metaph.* passages to Antisthenes at all strong.

9 Xenocrates at least once indulged in a logical circle (fr. 76 H); and he also tried to demonstrate definitions (*APo.* B 4, 91^a 35 - ^{b} 11: see Barnes, *APA*, p. 200). Cherniss, *ACPA*, pp.67-8, suggested that 'it was from the practice, if not from the express doctrine of Xenocrates, that the belief in universal demonstrability ἐξ ἀλλήλων took its rise'. (Cf. H.F. Cherniss, 'Plato as Mathematician', *Review of Metaphysics*, 4 (1950/1), 395-425, at p.417, n.50 (hereafter *PM*).) Ross, *APPA*, p.514, follows Cherniss; but the conjecture is evidently frail. I have not come across any other attempt to put a name to οἱ δέ.

10 See J. Barnes, 'Aristotle's Theory of Demonstration', *Phronesis*, 14 (1969), 123-52, at pp. 127-32 – further references are given there; see also I. Mueller, 'Greek Mathematics and Greek Logic', in *Ancient Logic and its Modern Interpretations*, ed. J. Corcoran (Dordrecht, 1974).

11 See R. Robinson, 'Analysis in Greek Geometry', *Mind* 45, (1936), 464-73 (repr. in his *Essays in Greek Philosophy* (Oxford, 1969)), at p.465; M.S. Mahoney, 'Another Look at Greek Geometrical Analysis', *Archive for History of Exact Sciences* 5 (1968/9), 318-48, at p.326; J. Hintikka and U. Remes, *The Method of Analysis* (Dordrecht, 1974), pp.37-8.

12 The interpretation of this passage is difficult; see Heath, *MA*, pp. 27-30; id., 'On an Allusion in Aristotle to a Construction for Parallels', *Abhandlungen zur Geschichte der Mathematik* 9 (1899), 155-60; I. Toth, 'Das Parallelenproblem im Corpus Aristotelicum', *Archive for History of Exact Sciences* 3 (1966/7), 249-422, at pp. 256-74.

These three straws will not by themselves make any bricks; but they do encourage us to look for a little clay. If some geometers did hit upon the notion of circular proof, how and why did they do so?

(b) *Geometrical Analysis*: Aristotle mentions the convertibility of mathematical propositions in connection with that most celebrated heuristic device, the 'method of analysis' (*APo.* A 12, 78^a 6–10). Pappus describes the method as follows:

> Analysis is a way from what is sought as being admitted through the things that come next in order to something admitted by synthesis. For in the analysis, hypothesizing what is sought as having come about, we consider the thing from which it follows, and again the antecedent of that, until, retreating in this way, we arrive at one of the things that are already known or that have the rank of a principle. . . . In the theoretical kind <of analysis> , hypothesizing what is sought as being the case and as true, and then proceeding to something admitted through the things that come next in order as true and as being the case hypothetically, if the thing admitted is true, what is sought will be true too, and the demonstration will be complementary to the analysis; and if we light upon something admitted to be false, what is sought will be false too (*Collectio,* 634.10–6 H; 634,25 - 636.7 H).

The interpretation of this text is controversial;[13] nor is it certain that Pappus' description is true of the analyses with which Aristotle was familiar: the following paragraphs are, therefore, tentative.

The analytical geometer examines a putative theorem, T. He considers first the 'things that come next in order', or the logical consequences of T; and he selects from among them 'the thing from which it follows', or some proposition which entails T. This proposition, Q_1, is then subjected to the same treatment as T; and it yields Q_2. The process is reiterated; and it ends, if the analysis is successful, by producing, as Q_n, a geometrical axiom. Analysis thus generates a sequence of propositions:

$$D = \langle T, Q_1, Q_2, \ldots Q_n \rangle$$

An act of 'synthesis' inverts the order of D and thereby presents a formal demonstration of T.

By construction, all the members of D are logically equivalent: the analysis infers Q_n from T, and the synthesis T from Q_n. The temptation to construe this in terms of circular proof is considerable: the geometer, applying the method of analysis, has apparently first proved Q_n on the basis of T and then T on the basis of Q_n; and in the course of his double journey no member of D has not been proved.

The method of analysis connects with Plato's account of mathematical method in the *Republic,* 511 b;[14] and I see no reason to doubt the old story that Plato particularly recommended the method of analysis to the Academic mathematicians.[15] An Academic geometer versed in the analytic method might have con-

[13] Pappus' text is printed in I. Thomas (ed.), *Greek Mathematical Works* (London, 1939 – Loeb Classical Library), ii. 596–8; see also pseudo-Euclid, in *Euclidis Opera,* iv.364–6 H. On the modern controversy see esp.: Heath, *E* i. 137–42; F.M. Cornford, 'Mathematics and Dialectic in the Republic', *Mind* 41 (1932), 37–52 and 173–90; Robinson, op.cit., N. Gulley, 'Greek Geometrical Analysis', *Phronesis,* 3 (1958), 1–14; Mahoney, op. cit.; Hintikka and Remes, op.cit. (see my review in *Mind* 85 (1976)).

[14] See Cornford, op. cit., pp. 43–50; *contra* R. Robinson, *Plato's Earlier Dialectic*[2] (Oxford, 1953), p.166.

[15] See Proclus, *Comm. in Eucl.* 211. 18–22; *Acad.Ind.Herc.* 15–7 M; Diogenes Laertius, 3. 24 (= T 17–18b in K. Gaiser, *Platons ungeschriebene Lehre* (Stuttgart, 1963)). C. Mugler, *Platon et la recherche mathématique de son époche* (Strassburg/

structed his twofold arguments as circular proofs and then have wondered whether circular proof was not the paradigm of scientific reasoning; and an Academic geometer acquainted with Academic epistemology might have surmised that the theory of circular proof threw a lifebelt to demonstrative science as it floundered in the regressive seas of scepticism.

(c) *An Academic Geometer*: 'Amyclas of Heracleia, one of Plato's followers, Menaechmus, a student of Eudoxus who also was associated with Plato, and his brother Deinostratus, made the whole of geometry more perfect.'[16] Menaechmus, who now becomes the hero of my tale, was thus both an Academician and a geometer – one of those mathematical moths who burned their wings in the candle of Platonism.[17]

Menaechmus' geometrical achievements were considerable. According to an honourable tradition,[18] one of the problems set for the Academic mathematicians was that of the duplication of the cube – the 'Delian problem' of constructing a cube twice the volume of a given cube.[19] Hippocrates had reduced this problem to the problem of constructing a pair of mean proportionals:[20] Menaechmus discovered the three conic sections (parabola, hyperbola, ellipse),[21] and was able to apply his discovery to the construction of mean proportionals (Proclus, *Comm. in Tim.* 34.2 D = fr. 9 Sch.).

Menaechmus' work is accounted one of the minor glories of Greek mathematics. Two constructions of the mean proportionals have been preserved under

Zurich, 1948), ch.5, argued that Plato invented the method of analysis; Cherniss's *PM* is a long review of Mugler, and it disputes his ch. 5 at pp. 414–9. But I still cannot see good reason to doubt that Plato applauded the method of analysis and commended it to his Academic geometers.

16 Proclus, *Comm. in Eucl.* 67.8–12 = Menaechmus, fr. 4 Sch. (see M.C.P. Schmidt, 'Die Fragmente des Mathematikers Menaechmus', *Philologus*, 42 (1884), 72–81). Here and hereafter I use Morrow's translation of Proclus: G.R. Morrow, *Proclus – A Commentary on the First Book of Euclid's Elements* (Princeton, 1970). On Dinostratus see G.J. Allman, *Greek Geometry from Thales to Euclid* (Dublin, 1889), pp.180–93; M. Cantor, *Vorlesungen über Geschichte der Mathematik*² (Leipzig, 1894), i. 233–4; G. Loria, *Le scienze esatte nell' antica Grecia*² (Milan, 1914), pp. 160–4; B.L. van der Waerden, *Science Awakening* (Groningen, 1954), pp.191–3.

17 The texts bearing on this are collected in Gaiser, op.cit., pp.460–74. Cherniss, *PM*, is still the best survey of the texts bearing on Plato's attitude to contemporary mathematics; see also T.L. Heath, *A History of Greek Mathematics* (Oxford, 1921), i. 284–315 (hereafter *GM*).

18 See pseudo-Eratosthenes in Eutocius, *Comm. in Arch.* 3. 88.5 – 90.29 H (see esp. E. Hiller, *Eratosthenis Carminum Reliquiae* (Leipzig, 1872), pp.122–37). Other texts are referred to by Gaiser, op.cit., p.474.

19 See e.g. Heath, *GM* i. 244–70; M. Simon, *Geschichte der Mathematik im Altertum* (Berlin, 1909), pp.192–203; E.P. Wolfer, *Eratosthenes von Kyrene als Mathematiker und Philosoph* (Groningen, 1954), pp.4–12; R. Böker, 'Würfelverdopplung', *RE* IXA 1 (1961), 1193–1223. Texts in Thomas, op.cit. i. 256–308; a modern presentation in H. Eves, *An Introduction to the History of Mathematics*³ (New York, 1969), pp.82–4.

20 See Pseudo-Eratosthenes in Eutocius, *Comm. in Arch.* 3, 88.17–23 H. The proof is simple: Let x, y, be mean proportionals between a, b; i.e. let $a : x = x : y = y : b$. Then $ay = x^2$, $bx = y^2$, $xy = ab$. Hence $x = y^2/b$; and $ay = y^4/b^2$. Thus $ab^2 = y^3$. Now let $a = 2b$. Then $2b^3 = y^3$. Thus if b is the side of the given cube, then y is the side of its double.

21 See Eratosthenes in Proclus, *Comm. in Eucl.* 3.20–3 = Menaechmus, fr. 3 Sch; more fully in Eutocius, *Comm. in Arch.* 3. 96, 10–27 (see Hiller, op.cit., p.130; cf. Eudoxus, D 24 L). On Menaechmus' discovery see Cantor, op.cit. i. 231–3; Heath, *GM* ii. 110–6; O. Neugebauer, 'The Astronomical Origin of the Theory of Conic Sections', *Proceedings of the American Philosophical Society* 92 (1948), 136–8.

his name (Eutocius, *Comm. in Arch.* 3. 78-13 – 84.7 H = fr. 11 Sch.). They have been much discussed.[22] My interest here is in their presentation, not in their content; for each applies, in a fully explicit form, the analytic method: each construction consists of a symmetrical pair of reasonings, analysis and synthesis. The wording of Eutocius is not original;[23] but there is no reason to deny that the form of the constructions goes back to Menaechmus himself.

Menaechmus was not merely a good technician. The meagre evidence reveals a mathematician interested in the philosophy and theory of his discipline.[24] One report shows a debate among the Academics about the status of mathematical truth: is the mathematician a discoverer, laying bare the structure of a world that exists independently of him and his cerebrations? Or is he rather an inventor, whose researches create a new realm of reality from the inner resources of his own mind?[25] The debate still thrives. Amphinomus and Speusippus, we are told, were discoverers or Platonists, and they consequently insisted on calling mathematical truths *θεωρήματα* or objects of contemplation; Menaechmus was an inventor or constructivist,[26] and he preferred the term *προβλήματα* (Proclus, *Comm. in Eucl.* 77.7 - 79.2 = fr. 6 Sch. = Speusippus, fr. 46 L).

Amphinomus was one of those geometers who denied the occurrence of 'causation' in mathematics. And on another matter, yet closer to the issue of circular proof, he is paired with Menaechmus: both men, according to Proclus, were interested in the phenomenon of conversion, and had a keen eye for the dangers of fallacious *ἀντιστροφαί* (*Comm. in Eucl.* 235.15 - 254.6 = fr. 7 Sch.).[27]

Here is an Academic geometer, given to the use of analysis, who has a sharp interest in the theory of knowledge: an easy conjecture makes Menaechmus the author of the theory of circular proof.

(d) *Menaechmus, fr. 5 Sch.*: The conjecture seems to be confirmed by a remarkable passage from Proclus' commentary on Euclid:

[22] See C.A. Bretschneider, *Die Geometrie und die Geometer vor Euclides* (Leipzig, 1870), pp.155-63; J. Gow, *History of Greek Mathematics* (Cambridge, 1884), pp.185-8; Allman, op.cit. pp.153-79; Cantor, op.cit. i. 217-18; T.L. Heath, *Apollonius of Perga – Treatise on Conic Sections* (Cambridge, 1896), pp. xvii-xxx; (hereafter *ATC*); H.G. Zeuthen, *Geschichte der Mathematik im Altertum und Mittelalter* (Copenhagen, 1896), pp.191-9; Loria, op.cit., pp.149-57; Heath, *GM* i. 252-5; Simon, op.cit., pp. 208-10; F. Kliem, 'Menaichmos', *RE* xv. 1, 1931, 700-1; Mugler, op.cit., pp.324-8; P.H. Michel, *De Pythagore à Euclide* (Paris, 1950), pp.252-3; van der Waerden, op.cit., pp.162-5; Böker, op.cit., cols.1211-13; F. Lasserre, *The Birth of Mathematics in the Age of Plato* (London, 1964), pp.119-23.

[23] See Gow, op.cit., p.186, n.3; Allman, op.cit., p.165; Loria, op.cit., p.153, n.2.

[24] And he had an independent interest in philosophy, if he is identical with *Μάναιχμος · Ἀλωπεκοννήσιος, κατὰ δέ τινας Προικοννήσιος, φιλόσοφος Πλατωνικός. ἔγφαψε φι λόσοφα καὶ εἰς τὰς Πλάτωνος πολιτείας βιβλία γ'* (Suda, s.v. *Μάναιχμος*). For the identification see T.H. Martin, *Theonis Smyrnaei Liber de Astronomia* (Paris, 1849), pp.58-60; Allman, op.cit., p.153, n.1; Loria, op.cit., p. 149, n.2; Michel, op.cit., p.252. Against: Bretschneider, op.cit., p.162; Schmidt, op.cit., pp.77-8.

[25] For Aristotle's view see *Cael.* 279^b 32 - 280^a 10 = Speusippus, fr. 54 a L. See, e.g., E. Niebel, *Untersuchungen über die Bedeutung der geometrischen Konstruktion in der Antike*, Kantstudien, Ergänzungshefte 76 (Köln, 1959), pp.89-103.

[26] See esp. Zeuthen, op.cit., pp.88-91; cf. O. Becker, *Grundlagen der Mathematik*[2] (Freiburg/Munich, 1964), pp.90-1.

[27] Amphinomus is mentioned a fourth time by Proclus as having discussed different types of *προβλήματα* (*Comm. in Eucl.* 220.9).

The term 'element' (στοιχεῖον) can be used in two senses (διχῶς λέγεται),[28] as Menaechmus tells us. For what proves (τὸ κατασκευάζον) is called an element of what is proved by it. Thus in Euclid the first theorem is an element of the second, and the fourth of the fifth. In this sense many propositions can be called elements of one another, when they can be established reciprocally (κατασκευάζεται . . . ἐξ ἀλλήλων). From the proposition that the exterior angles of a rectilinear figure are equal to four right angles we can prove the number of right angles to which the interior angles of the figure are equal,[29] and vice versa. An element so regarded is a kind of lemma (λῆμμα). But in another sense 'element' means a simpler part into which a compound can be analysed. In this sense not everything can be called an element of anything [that follows from it],[30] but only the more primary members of an argument leading to a conclusion (τὰ ἀρχοειδέστερα τῶν ἐν ἀποτελέσματος λόγῳ τεταγμένων), as postulates (αἰτήματα) are elements of theorems (*Comm. in Eucl.* 72.23 – 73.9 = fr. 5 Sch.).

This is not a verbatim quotation from Menaechmus – the reference to Euclid is enough to show that.[31] It is tempting, indeed, to hear some Menaechman echoes in Proclus' words: thus κατασκευάζειν regularly means 'construct' in geometrical texts,[32] and its use here for 'prove' admirably fits Menaechmus' constructivist bent; again, αἴτημα appears to be used here as a general term for axioms, and that usage is associated by Proclus with the usage of πρόβλημα as a general term for mathematical truths (*Comm. in Eucl.* 179.8–12).[33] But κατασκευάζειν is a normal word for 'establish' in Aristotle's dialectic (e.g. *Top.* 102ª 15; 109ᵇ 26; *APr.* 43ª 15); and the implications of Proclus' remark about αἰτήματα are not clear (in any case αἴτημα in fr. 5 Sch. is paired with the un-Menaechman word θεώρημα). Yet even if fr. 5 Sch. is wholly paraphrastic, it is in all probability a reliable paraphrase: the motif of διχῶς λέγεται is thoroughly Aristotelian; interest in the nature of στοιχεῖα is a known feature of life in the Academy; and the theorem on exterior and interior angles, though absent from Euclid, was familiar to Aristotle.[34]

The importance of the fragment is plain: it confirms the suggestion that Menaechmus advocated circular reasoning. In one sense of 'element', he allows, 'not everything can be called an element of anything'. But in another and perfectly respectable sense elements themselves can be proved: 'many propositions can be called elements of one another'; theorems can be established 'reciprocally' or circularly; and there can be demonstration of everything – of all the truths of geometry without exception. Menaechmus stands revealed as the first begetter of circular demonstration.

(e) *More About Menaechmus*: The few remaining references to Menaechmus do not shed any further light on his notion of proof; but they do connect him a

[28] On στοιχεῖον see esp. W. Burkert, 'CTOIXEION', *Philologus* 103 (1959), 167–97 (for Menaechmus see pp.191–6); for Aristotle's analysis of the term see *Metaph.* Δ. 3.

[29] At 73.3 the text is corrupt, but the sense is not in doubt. (The simplest emendation is perhaps excision of ὀρθαῖς ἴσων.)

[30] Morrow's insertion.

[31] See, e.g., Burkert, op.cit., p.191. Malcolm Brown is inclined to see the bulk of fr. 5 Sch. as a genuine quotation from Menaechmus. I am sceptical: some of the terminology (e.g. ἀποτέλεσμα) is late.

[32] See C. Mugler, *Dictionnaire historique de la terminologie géométrique des Grecs* (Paris, 1958), pp.245–6 (hereafter *DG*).

[33] Cf. Speusippus, fr. 30 L; and see P. Lang, *de Speusippi Academici Scriptis* (Bonn, 1911), pp.28–9; Lasserre, op.cit., pp.31–2 (though I cannot see that the use of αἴτημα implies that mathematics is reduced to 'an exercise in logic').

[34] *APo.* A 24, 85ᵇ 38; B 17, 99ª 19; see Heath, *MA*, pp. 62–4. I guess that the theorem was proved by Menaechmus.

little more closely with Aristotle, and they are in any event worth running quickly through.

First, there are two or three points of terminology. Fr. 5 Sch. begins, as I have already noted, with an Aristotelian phrase; in fr. 6 the language is reminiscent of the Aristotelian categories (Menaechmus says that sometimes *πεποριςμένον λαβόντας ἰδεῖν <δεῖ> ἢ τί ἐστιν ἢ ποῖόν τι ἢ τίνας ἔχει πρὸς ἄλλο σχέσεις*: Proclus, *Comm. in Eucl.* 78.10–13);[35] and the language of fr. 7 is wholly Aristotelian (one of the things which did not escape Menaechmus was that conversion holds *ἐφ' ὧν . . . τὸ πρώτως ὑπάρχον καὶ τὸ ᾗ αὐτὸ λαμβάνεται*: Proclus, *Comm. in Eucl.* 25.2). These connections between Aristotle and Menaechmus are, however, tenuous: they suppose the accuracy of Proclus' *reportage*; and if Proclus copied from Geminus, Geminus from Eudemus, and Eudemus from Menaechmus, the probability that we possess traces of Menaechmus' actual words in these brief fragments cannot be very high.[36]

Doctrine rather than language conjoins Menaechmus and Aristotle in astronomy and in mechanics. In astronomy both men adhered to a Eudoxan model of the cosmos (Theo Smyrn. *Expos. rer. math.* 201.22 - 202.7 = fr. 8 Sch.); and mechanics was a subject which, despite Plato's disapproval, Menaechmus engaged in[37] and Aristotle accepted as a science (*APo.* A 9, 76^{a} 24; A 13, 78^{b} 37). But again, there is nothing much in this.

The final testimony to Menaechmus deserves more attention. Stobaeus relates the following anecdote: 'Alexander asked Menaechmus the geometer to teach him geometry in a concise way. He said: "Your Majesty, on land there are common roads and royal highways; but in geometry there is one road for all" ' (2.31 = fr. 1 Sch.). The same edifying story is told of Euclid and Ptolemy Euergetes (Proclus, *Comm. in Eucl.* 68. 13–17). Euclid may have been its original hero; but I find it more plausible to suppose that it was first told of

[35] I accept O. Becker's palmary conjecture of *πεποριςμένον* for the manuscript reading *περιωρισμένον*. Friedlein prints *τίς ἐστιν* by mistake for *τί ἐστιν*.

[36] On Proclus' sources see P. Tannery, *La Géométrie grecque* (Paris, 1887), pp.71–5; Heath, *GM* i. 35–8; *E* i. 29–45; F. Wehrli, *Eudemos von Rhodos, Die Schule des Aristoteles* 8 (Basel, 1955), pp.114–15.

[37] The question is complicated. Plutarch, *Quaest.conviv.* 718 F (= Menaechmus, fr. 2 Sch. = Eudoxus, D 28 L = T 21a in Gaiser, op.cit.) says that Plato reproached Eudoxus, Archytas, and Menaechmus *εἰς ὀργανικὰς καὶ μηχανικὰς κατασκευὰς τὸν τοῦ στερεοῦ διπλασιασμὸν ἀπάγειν ἐπιχειροῦντας* (cf. Plutarch, *Vit.Marc.* 14.5 = Eudoxus, D 27 L = T 21b in Gaiser, op.cit.). Ps.-Eratosthenes reports of *τῶν . . . ζητούντων δύο τῶν δοθεισῶν δύο μέσας λαβεῖν* that *συμβέβηκε . . . πᾶσιν αὐτοῖς ἀποδεικτικῶς γεγραφέναι, χειρουργῆσαι δὲ καὶ εἰς χρείαν πεσεῖν μὴ δύνασθαι πλὴν ἐπὶ βραχύ τι τὸν Μεναίχμον, καὶ ταῦτα δυσχερῶς* (Eutocius, *Comm. in Arch.* 3. 90.4–11 H = Menaechmus, fr. 10 Sch. = Eudoxus, D 25 L). These passages suggest the following story: Eudoxus and his friends, having provided a geometrical solution to the Delian problem, attempted to apply their solution to the practical construction of a cube. Their attempt was unsuccessful – and it aroused the intellectual disapproval of Plato. (See esp. J.L. Heiberg, 'Jahresbericht – griechische und römische Mathematik', *Philologus,* 43 (1884), 467–522, at p.475; van der Waerden, op.cit., pp.163–5.) Very different accounts have, however, been canvassed (e.g. Schmidt, op.cit., pp.78–80; Allman, op.cit., pp.170–2; Heath, *ATC*, pp. xxix-xxx; Niebel, op.cit., pp.112–33); and the continuation of Plutarch's text is in any case corrupt. The commonplace that Menaechmus 'engaged in mechanics', which I repeat in the text, is thus neither very clear in meaning nor very well supported by the ancient evidence (see Tannery, op.cit., pp.79–80, who is sceptical of the whole story). On the history of mechanics from Plato to Archimedes see esp. F. Solmsen, *Die Entwicklung der aristotelischen Logik und Rhetorik* (Berlin, 1929), pp.130–5 (hereafter *ELR*).

Menaechmus and then transferred from him to his more famous successor.[38] That does not matter; nor, in any case, is the story more than *ben trovato.* What matters is that the author of the anecdote believed that Menaechmus had been tutor to Alexander the Great. We have no reason to deny the truth of his belief; and we know that Aristotle was also tutor to Alexander. It is reasonable to infer that Menaechmus and Aristotle were for some years colleagues and fellow tutors at the court of Pella.[39]

(f) *A Simple Story*: The evidence invites an uncomplicated conclusion. The theory of circular proof was first hit upon by Menaechmus, Aristotle's friend and colleague, in the course of his geometrical researches. Menaechmus' philosophical interests led him to apply the theory to a standing problem in epistemology, the sceptical regress. When Aristotle came to consider the regress, he devoted much thought to his friend's attempt to evade it; but his own theory of demonstrative knowledge obliged him to reject his friend's solution – here too, *magis amica veritas.*

III. Aristotle

(a) *A Little Cold Water*: I have elicited an account of circular proof from Menaechmus, fr. 5 Sch. Consider the simplest case in which there are two propositions, P_1, P_2, and two sound arguments: P_1 so P_2; P_2 so P_1. According to my interpretation of fr. 5, there is one sense of 'element' (call it the Aristotelian sense) in which P_1 and P_2 cannot both be elements; but there is another sense (the Menaechman sense) in which they can. And given this latter sense, each of the two sound arguments involving P_1 and P_2 can be regarded as a proof.

Now Proclus' illustrations of Menaechmus' view do not fit this interpretation with any accuracy. For it is certainly not the case that Euclid's first theorem, T_1, entrails his second, T_2: in Euclid, T_2 is not derived from T_1 alone; nor can it be so derived. Again, the two theorems on the angles of a polygon – T_3 and T_4 – are not mutually interderivable: T_4 is not deducible from T_3 alone, nor T_3 from T_4. Rather, the geometrical facts are these: where A is some subset of Euclid's axioms, we have the following three sound arguments: A and T_1, so T_2; A and T_3, so T_4; A and T_4; A and T_4, so T_3.

These facts were surely known to Menaechmus: did he (or Proclus) simply offer an infelicitous illustration of his view? or is my interpretation of his view wrong? A closer inspection of fr. 5 gives preference to the latter alternative. First, T_3 and T_4 can be said, with perfect propriety, to be 'established reciprocally'; for each theorem figures as a premiss in a sound argument for the other. Secondly, in the Aristotelian sense of 'element' the elements are said to be like 'postulates' or axioms; and in this sense A, and A alone, is an element in the arguments for T_2, T_3, and T_4. Thirdly, it is only 'the *more* primary members of an argument leading to a conclusion' which are elements in the Aristotelian sense: the use of the comparative adjective, ἀρχοειδέστερα, points

[38] See Schmidt, op.cit., p.78; Allman, op.cit., p.154; Loria, op.cit., p.149, n.3; Heath, *GM* i. 252; van der Waerden, op.cit., p.190. *Contra*: Bretschneider, op.cit., p.163.

[39] On Alexander's tutors see the texts assembled in I. Düring, *Aristotle in the Ancient Biographical Tradition* (Göteborg, 1957), pp.284–99; cf. W. Jaeger, *Aristotle*² (Oxford, 1948), pp.120–3; A.H. Chroust, *Aristotle* (London, 1973), i. 125–32.

clearly to a distinction between different types of premiss occurring in a single argument.

Thus fr. 5 Sch. does not say that, given one sense of 'element', reciprocal demonstration is legitimate. Rather, it says something like this: 'the propositions cited in the course of a Euclidean proof are all, in a broad sense, elements of the theorem proved; but some of these elements are also elements in the narrower sense of being undemonstrable axioms.' In short, Menaechmus states that '*P* is an element for *Q*' may mean either '*P* is a proposition used in a proof of *Q*' or '*P* is an axiom used in a proof of *Q*'.[40]

Then does fr. 5 have anything to say about circular proof? Menaechmus allows that both 'A, T_3 : so T_4' and 'A, T_4 : so T_3' may count as proofs; but he does not say that *A* itself can be proved, nor does he say that geometry can do without elements in the Aristotelian sense. Thus he admits some demonstrative reciprocity; and that admission is sufficient to set him at odds with the view Aristotle takes in *APo.* A 3. But it is not sufficient to ground a theory of circular proof full-blooded enough to escape the sceptical regress. To do that, Menaechmus would have to admit 'T_3, T_4 : so A' alongside his proofs of T_3 and T_4.

(b) *Syllogistic Circles*: The direct link between Menaechmus and *APo.* A 3 snaps. But perhaps we can forge a longer and stronger chain. *APo.* A 3 refers (73^a 14) to *APr.* B 5-7 for a detailed discussion of the logic of circularity. There Aristotle gives a formal definition of circular proof: 'To prove circularly and reciprocally is to take the conclusion and the one premiss (with its predication reversed) and to conclude to the remaining premiss which was assumed in the other syllogism' (57^b 18–21; cf. 58^a 34–5). Take a syllogism '*AxB*, *ByC*: so *AzC*'; first convert *AxB* to *BxA* and form the syllogism '*AzC*, *BxA*: so *ByC*'; then convert *ByC* to *CyB* and form the syllogism '*AzC*, *CyB*: so *AxB*'. (*APr.* B 5–7 shows that perfect circles of this sort are possible only in *Barbara*, where we may have: *AaB*, *BaC*: so *AaC*; *AaC*, *BaA*: so *BaC*; *AaC*, *CaB*: so *AaB*.[41] (Circular proof evidently requires convertibility (73^a 7): let a science *S*† consist of the propositions *AaB*, *BaA*, *BaC*, *CaB*, *AaC*, *CaA*. Then by way of circular syllogistic every member of *S*† can be proved.

Aristotle presents his circles in formal syllogistic dress; and that dress fits Euclidean argumentation ill. But Aristotle himself thought that syllogistic could be used to formalize geometrical argument; and his syllogistic circles are clearly connected with the reciprocities which Menaechmus had observed in certain geometrical proofs. I imagine that Menaechmus brought his reciprocal proofs of T_3 and T_4 to Aristotle's notice. Aristotle assumed that these proofs could be represented within his syllogistic; and he believed that mathematical propositions were in general convertible. He then investigated the conditions under which reciprocity was syllogistically possible; and his logical inquiries led him to complete the circles which Menaechmus had left half drawn. Menaechmus'

[40] In the Menaechman sense an element is 'like a lemma': elsewhere (*Comm. in Eucl.* 211.5) Proclus defines 'lemma' as 'a proposition requiring proof'; but in fr. 5, the word has its more general sense of 'assumption', λαμβανόμενον (see Mugler, *DG*, pp. 261–2).

[41] *AaB* represents '*A* belongs to every *B*' or 'Every *B* is *A*' (for this way of writing syllogistic propositions see G. Patzig, *Aristotle's Theory of the Syllogism* (Dordrecht, 1969), pp.49–50). *x*, *y*, *z*, are variables ranging over the syllogistic relations *a*,*e*,*i*,*o*. On syllogistic circles see further Barnes, *APA*, p.108.

two inferences, 'A, T_3: so T_4' and 'A, T_4: so T_3', were assimilated to the two syllogisms '*AaB*, *BaC*: so *AaC*' and '*AaC*, *BaA*: so *BaC*'. Aristotle added '*AaC*, *CaB*: so *AaB*', which he took to represent 'T_3, T_4: so A'.

(c) *Arguments and Proofs*: Roughly speaking, *APr.* is concerned with validity, *APo.* with proof. Yet in some passages *APr.* narrows its concerns. B 5–7 insists that circular arguments require convertible propositions (57^b 30 - 58^a 15); and the insistence shows that Aristotle is not concerned solely with the validity of the arguments which make up his circles. For the syllogism '*AaC*, *BaA*: so *BaC*' is valid whether or not *AaB* converts – it is an argument in *Barbara*, and all such arguments are valid. The convertibility of *AaB* is relevant only if Aristotle is concerned with the soundness of his circular syllogisms; for if '*AaB*, *BaC*: so *AaC*' is sound, then '*AaC*, *BaA*: so *BaC*' is sound if and only if *AaB* converts.

Now sound arguments, as Aristotle thoroughly knew (*APo.* A 13), are not necessarily proofs. But the language of proof is ubiquitous in *APr.* B 5–7: in B 5 alone, δείκνυσθαι and its cognates occur some eighteen times, ἀποδείκνυσθαι and its cognates some ten times – and these are Aristotle's official words for 'prove' and 'demonstrate'. If we take *APr.* B 5–7 at its word, then it, like *APo.* A 3, is concerned with the theory of proof or demonstration; and an outrageous conclusion begs to be drawn: *APr.* B 5–7 expounds and advocates a theory of circular proof; the coyly anonymous οἱ δέ of *APo.* A 3 can be given a name – 'Aristotle'.

Such a suggestion is highly unpalatable: perhaps the language of proof is used loosely in *APr.* B 5–7; perhaps δείκνυσθαι and ἀποδείκνυσθαι mean merely 'provide a sound argument for'. Why *must* we take them in their stronger, official sense? I admit that Aristotle often uses δείκνυσθαι where 'prove' is less apposite than 'argue soundly', or even 'argue validly'; and I grant that his logical vocabulary, being poverty-stricken, is given to promiscuity. Nevertheless, I am reluctant to ignore the official sense of (ἀπο-) δείκνυσθαι; and I believe that *APr.* B 5–7 at least makes it appropriate to ask whether any other Aristotelian passages support the thesis that Aristotle himself was once a circular theorist.

(d) *The Cycles of Nature*: The old Greek idea that the world exhibits cycles of generation and decay had a deep hold on Aristotle. Fundamental to his picture of the physical world are two cyclical changes: the orbits of the heavenly bodies, and the regular interchanges of the four primary elements. Whatever is brought about by circular motion itself must proceed along a circular path; and these two cycles are responsible for all natural phenomena: thus, in a sense, circularity pervades the whole of Aristotle's world.[42]

The *locus classicus* for all this is the end of the *de Generatione et Corruptione*: I quote two short passages from the long and involved argument of B 10–11.

... God perfected the universe by making generation continual; for in this way being would acquire the greatest coherence, because the eternal generation of generation is as near as possible to being. And the cause of this, as has often been said, is circular locomotion; for this alone is continuous. That is why all the other things that change into one another by virtue of their qualities and their powers (e.g. the simple bodies) imitate circular locomotion; for whenever air comes from water, and fire from air, and water again from fire, we say that

[42] References in Bonitz, *Index Aristotelicus*, 414^a 6–14; see esp. H.H. Joachim, *Aristotle on Coming-to-be and Passing-away* (Oxford, 1922), pp.254–66; F. Solmsen, *Aristotle's System of the Physical World* (Ithaca, 1960), pp.420–39. On the idea of cycles of generation see also C. Mugler, *Deux thèmes de la cosmologie grecque* (Paris, 1953).

the generation has come round in a circle (κύκλῳ . . . περιεληλυθέναι) because it reciprocates back (πάλιν ἀνακάμπτειν) (336^b 30 - 337^a 7).

If what moves in a circle (κύκλῳ) always moves something, necessarily the motion of these things too is circular – e.g. since the upper locomotion is circular, the sun moves in this way; and if that is the case, for this reason the seasons come about in a circle and reciprocate (ἀνακάμπτουσι); and if these come about in this way, the things under them again will do so (338^b 1–5).

There is the closest connection between *GC* B 10–11 and the *Analytics*. The chapters make free use of logical terminology: the syllogistic formula ἀνάγκη τούτων ὄντων occurs at 336^a 16; ἐξ ὑποθέσεως appears at 337^b 27; ἐπὶ τὸ κάτω is at 337^b 26 (cf. *APo.* A 19, 82^a 2 etc.); more significantly, ἀντιστρέφειν is used twice (337^b 24; 338^a 11), and ἀνακάμπτειν six times (337^a 7; 338^a 5; ^{b} 5; 9; 14; 17). Moreover, between the two works there are several substantial similarities, both of form and of content: compare 338^a 14 with 73^a 36–7; 337^a 25–34 with 95^b 1–12; 337^b 1 with 95^b 13; 337^b 14–25 with 95^b 31–37; 338^b 7–8 with 96^a 2–5; 337^a 6 with 96^a 6.

It is a natural thought that Aristotle may have hoped to order a cyclical nature by means of a circular logic; for surely the ἀνάκαμψις observable throughout the physical world will be reflected in that bright mirror of nature, the demonstrative syllogism? And surely the reflexion will assume the form of a logical ἀνάκαμψις?

(e) *Nature and Logic*: Of the six substantial similarities I listed between *GC* B 10–11 and the *Analytics*, one relates to *APo.* A 3 and the other five to *APo.* B 12. The 'natural thought' inspired by *GC* B 10–11 is, I think, confirmed by *APo.* B 12. An appendix at the end of that chapter runs as follows:

Since we see that among the things that come about there is a sort of circular coming about (κύκλῳ τινὰ γένεσιν), it is possible for this to be the case if the middle term and the extremes follow one another. For in these cases there is conversion (this has been proved in our first <chapters>), because the conclusions convert; and that is what being circular is. – In actual cases it appears as follows: if the earth is soaked, necessarily steam came about; and if that came about, cloud; and if that came about, water; and if that came about it is necessary for the earth to be soaked. But this was what we started from; so that it has come round in a circle (κύκλῳ περιελήλυθεν) – for if any whatever of them is the case, another is; and if that another; and if that, the first (95^b 38 - 96^a 7).

Both the argument and the illustration in this appendix are obscure.[43] I gloss the argument thus: 'Suppose we present a "circular" happening by way of the syllogism "*AaB*, *BaC*: so *AaC*". Now to say that *AaC* describes a "circular" happening is simply to say that, just as *A* follows on *C*, so *C* follows on *A*. Thus the conclusion converts to *CaA*. But if *AaC* converts in this way, then so too do *AaB* and *BaC*. (*BaC* and *CaA* yield *BaA*; *CaA* and *AaB* yield *CaB*.) Hence we have *AaB*, *BaA*, *BaC*, *CaB*, *AaC*, *CaA*.'

The illustration can be made to fit the syllogistic argument form by letting *A* stand for 'being an occasion when the earth is soaked', *B* 'being an occasion when the skies are clouded', and *C* 'being an occasion when rain falls'. (I omit steam for simplicity.) Thus we observe the cyclical interchange of rain and mud, expressing our observation as '*AaC* and *CaA*'. And then we explain *AaC* by *AaB* and *BaC*. We can then go on to explain *BaA* by *BaC* and *CaA*; *CaB* by *CaA* and *AaB*; *CaA* by *CaB* and *BaA*; *BaC* by *BaA* and *AaC*; and *AaB* by *AaC* and *CaB*.

43 See further Barnes, *APA*, pp.228–9. I punctuate the lines differently from Ross.

That illustration is, of course, fairly confused; and I am inclined to think that the sort of phenomena Aristotle is attempting to account for cannot be explained within the straitjacket of syllogistic: natural cycles cannot be driven by syllogistic chains. Nor, indeed, will an adequate formal account of cyclical change exhibit any *logical* circularity. But however that may be, the general thrust of Aristotle's argument in 95^b 38 –96^a 2 is unmistakable: circular processes in nature must, he supposes, be represented by circular arguments in logic. And the example confirms this: demonstrations in meteorology will be circular because the phenomena they explain are cyclical.

Now cyclical phenomena, according to Aristotle, are neither rare nor disreputable: the clear implication of *APo.* B 12 is that circular demonstrations are in themselves perfectly admissible, and that they will be admitted into the sciences just as often as the facts call for them. Circular proof, in short, is a legitimate logical device; and it is the scientist's only means of representing a certain pervasive type of natural event.

IV: Conclusion

The following account is offered as a plausible story of the history of circular proof: where truth is forever hidden verisimilitude is all that can be hoped for.

The Academic geometer Menaechmus observed that mathematical proofs often exhibited a certain limited reciprocity or circularity. He conveyed the fact to Aristotle who attempted to formalize and explain it within his syllogistic. Aristotle's work suggested to him that proofs might be wholly reciprocal in form; and he conjectured that a theory of circular proof would provide an answer to the sceptic's epistemological regress. A consideration of natural phenomena and their cyclical occurrence appeared to confirm the conjecture: the fundamental method of scientific demonstration was to be the circular syllogism.

More sober thought destroyed that happy theory. First, it became clear that in reality few propositions outside the realm of mathematics are convertible (*APo.* A 3, 73^a 17), and hence that the range of circular proof is severely restricted. Secondly, a nicer attention to the nature of scientific explanation ruled out entirely the possibility of circular demonstration: demonstrations essentially involve an asymmetrical relation. Aristotle's own youthful theory was thus overthrown;[44] and Aristotle replaced it by an account of scientific proof which was to survive in unchallenged glory for two millennia.[45]

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[44] If I am right, *GC* B 10–11 and *APr.* B 5–7 were written before *APo.* A 3; but that is hardly a startling hypothesis. (Solmsen holds, in *ELR*, p.145, n.2, that *APr.* B 5–7 recants *APo.* A 3: that is incredible, particularly in the light of 73^a 6–20; and Solmsen only suggests it because it follows from his general (and mistaken) theory that *APo.* was written before *APr.*)

[45] The final draft of this paper benefited from the critical scrutiny of my colleague, Robert Delahunty; an earlier version was read at a seminar organized by Professor David Balme, where it was the object of several helpfully sceptical animadversions. My major debt is to Malcolm Brown, of Brooklyn College: finding by chance that we shared an affection for old Menaechmus and believed him to have been an important influence on Aristotle, we planned to compose a joint paper. As our thoughts progressed, they diverged; and the conclusion of this paper is, alas, only mine. But that conclusion would not have been reached without Malcolm Brown's original enthusiasm and continued encouragement; and I thank him warmly.